РУСЕНСКИ УНИВЕРСИТЕТ "АНГЕЛ КЪНЧЕВ"
ФАКУЛТЕТ *ПРИРОДНИ НАУКИ И ОБРАЗОВАНИЕ*
КАТЕДРА *МАТЕМАТИКА*

ЗАДАЧИ

по

МАТЕМАТИЧЕН АНАЛИЗ II

за специалност

Компютърни науки

Русе

2015

1. Да се пресметнат неопределените интеграли:

1.1. $\int \left(3x^4 - 8x^3 + 5x^2 - 6x - 7\right)\,\mathrm{d}x.$ Отг. $\frac{3x^5}{5} - 2x^4 + \frac{5x^3}{3} - 3x^2 - 7x + C.$

1.2. $\int (x+1)(2x^2 - 3x + 3)\,\mathrm{d}x.$ Отг. $\frac{x^4}{2} - \frac{x^3}{3} + 3x + C.$

1.3. $\int \left(\frac{1}{x} + \frac{1}{x^2} - \frac{2}{x^3}\right)\,\mathrm{d}x.$ Отг. $\ln|x| - \frac{1}{x} + \frac{1}{x^2} + C.$

1.4. $\int \left(x^2 + \frac{3x}{5} + \frac{6}{x} - \frac{3}{x^4}\right)\,\mathrm{d}x.$ Отг. $\frac{x^3}{3} + \frac{3x^2}{10} + 6\ln|x| + \frac{1}{x^3} + C.$

1.5. $\int \left(2\sqrt{x} - \frac{1}{2}\sqrt[3]{x^2} + \frac{1}{\sqrt[4]{x^5}}\right)\,\mathrm{d}x.$ Отг. $\frac{4\sqrt{x^3}}{3} - \frac{3\sqrt[3]{x^5}}{10} - \frac{4}{\sqrt[4]{x}} + C.$

1.6. $\int \frac{\sqrt{x} - 2\sqrt[3]{x^2} + 1}{\sqrt[4]{x}}\,\mathrm{d}x.$ Отг. $\frac{4}{3}x^{\frac{3}{4}} + \frac{4}{5}x^{\frac{5}{4}} - \frac{24}{17}x^{\frac{17}{12}} + C.$

1.7. $\int \frac{3x^4 + 3x^2\mathrm{e}^x + x}{x^2}\,\mathrm{d}x.$ Отг. $x^3 + 3\mathrm{e}^x + \ln|x| + C.$

1.8. $\int \frac{1 + \cos^3 x}{\cos^2 x}\,\mathrm{d}x.$ Отг. $\operatorname{tg} x + \sin x + C.$

1.9. $\int \left(\operatorname{cotg}^2 x + 2\right)\,\mathrm{d}x.$ Отг. $-\operatorname{cotg} x + x + C.$

1.10. $\int \frac{1}{x^2(1+x^2)}\,\mathrm{d}x.$ Отг. $-\frac{1}{x} - \operatorname{arctg} x + C.$

1.11. $\int \frac{\cos 2x}{\cos^2 x \sin^2 x}\,\mathrm{d}x.$ Отг. $-\operatorname{cotg} x - \operatorname{tg} x + C.$

1.12. $\int \frac{x^4 + 2x + 4}{(x+2)x^4}\,\mathrm{d}x.$ Отг. $\ln|x+2| - \frac{2}{3x^3} + C.$

1.13. $\int \frac{x^2 + x + 6}{(x+2)(x^2+4)}\,\mathrm{d}x.$ Отг. $\ln|x+2| + \frac{1}{2}\operatorname{arctg}\frac{x}{2} + C.$

1.14. $\int \frac{(1+x)^2}{x(1+x^2)}\,\mathrm{d}x.$ Отг. $\ln|x| + 2\operatorname{arctg} x + C.$

1.15. $\int \left(2\cos 5x + \mathrm{e}^{7x} - \frac{1}{3x+2}\right)\,\mathrm{d}x.$ Отг. $\frac{2}{5}\sin 5x + \frac{1}{7}\mathrm{e}^{7x} - \frac{1}{3}\ln|3x+2| + C.$

1.16. $\int \left(\sqrt{x+2} + \frac{1}{\sqrt[5]{(2x-1)^2}}\right)\,\mathrm{d}x.$ Отг. $\frac{2}{3}(x+2)^{\frac{3}{2}} + \frac{5}{6}(2x-1)^{\frac{3}{5}} + C.$

1.17. $\int \left(\sin 4x + \cos(3x-5) + \mathrm{e}^{-x}\right)\,\mathrm{d}x.$ Отг. $-\frac{1}{4}\cos 4x + \frac{1}{3}\sin(3x-5) - \mathrm{e}^{-x} + C.$

1.18. $\int \left(\frac{1}{\sin^2 x.\cos^2 x} - \frac{1}{1+4x^2}\right)\,\mathrm{d}x.$ Отг. $\operatorname{tg} x - \operatorname{ctg} x - \frac{1}{2}\operatorname{arctg} 2x + C.$

1.19. $\int \frac{2x}{x^2+1}\,\mathrm{d}x.$ Отг. $\ln(x^2+1) + C.$

1.20. $\int \frac{2x}{(x^2+1)^4}\,\mathrm{d}x.$ Отг. $-\frac{1}{3(x^2+1)^3} + C.$

1.21. $\int \frac{\cos x}{\sin^5 x}\,\mathrm{d}x.$ Отг. $-\frac{1}{4\sin^4 x} + C.$

1.22. $\int \mathrm{e}^{2\sin x}\cos x\,\mathrm{d}x.$ Отг. $\frac{1}{2}\mathrm{e}^{2\sin x} + C.$

1.23. $\int \frac{\cos x}{1+2\sin x}\,\mathrm{d}x.$ Отг. $\frac{1}{2}\ln|1+2\sin x| + C.$

1.24. $\int \frac{\sqrt{\arctan x}}{x^2+1}\,\mathrm{d}x.$ Отг. $\frac{2}{3}(\operatorname{arctg} x)^{\frac{3}{2}} + C.$

1.25. $\int \frac{1}{x(\ln x+1)}\,\mathrm{d}x.$ Отг. $\ln|\ln|x|+1| + C.$

1.26. $\int \frac{\sin 2x}{1+\sin^2 x}\,\mathrm{d}x.$ Отг. $\ln(1+\sin^2 x) + C.$

1.27. $\int \frac{1}{1+3\operatorname{cotg} x}\cdot\frac{1}{\sin^2 x}\,\mathrm{d}x.$ Отг. $-\frac{1}{3}\ln|1+3\operatorname{cotg} x| + C.$

1.28. $\int \frac{\sqrt[3]{\operatorname{tg} x}}{\cos^2 x}\,\mathrm{d}x.$ Отг. $\frac{3}{4}\operatorname{tg}^{\frac{4}{3}} x + C.$

1.29. $\int \frac{2x+\operatorname{arctg}^2 x}{1+x^2}\,\mathrm{d}x.$ Отг. $\ln(x^2+1) + \frac{1}{3}\operatorname{arctg}^3 x + C.$

1.30. $\int \frac{\mathrm{e}^x}{\sqrt{1-\mathrm{e}^{2x}}}\,\mathrm{d}x.$ Отг. $\arcsin \mathrm{e}^x + C.$

1.31. $\int \frac{1}{\sin x\cos^2 x}\,\mathrm{d}x.$ Отг. $\ln\left|\operatorname{tg}\frac{x}{2}\right| + \frac{1}{\cos x} + C.$

1.32. $\int \frac{x+\sqrt{\arcsin x}}{\sqrt{1-x^2}}\,\mathrm{d}x.$ Отг. $-\sqrt{1-x^2} + \frac{2}{3}\arcsin^{\frac{3}{2}} x + C.$

1.33. $\int \sin^3 x\,\mathrm{d}x.$ Отг. $-\cos x + \frac{1}{3}\cos^3 x + C.$

1.34. $\int (\sin^2 x + \cos^2 2x)\,\mathrm{d}x.$ Отг. $x - \frac{1}{4}\sin 2x + \frac{1}{8}\cos 4x + C.$

1.35. $\int \cos^4 x \, \mathrm{d}x.$ Отг. $\frac{3x}{8} + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C.$

1.36. $\int x \, \mathrm{e}^{3x} \, \mathrm{d}x.$ Отг. $\left(-\frac{1}{9} + \frac{x}{3}\right)\mathrm{e}^{3x} + C.$

1.37. $\int x \, \cos 2x \, \mathrm{d}x.$ Отг. $\frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C.$

1.38. $\int (2x+1)\sin x \, \mathrm{d}x.$ Отг. $-(1+2x)\cos x + 2\sin x + C.$

1.39. $\int x^2 \, \sin x \, \mathrm{d}x.$ Отг. $-x^2\cos x + 2x\sin x + 2\cos x + C.$

1.40. $\int x^3 \ln x \, \mathrm{d}x.$ Отг. $\frac{x^4}{4}\ln|x| - \frac{x^4}{16} + C.$

1.41. $\int \frac{1}{x^3}\ln x \, \mathrm{d}x.$ Отг. $-\frac{1}{2x^2}\ln|x| - \frac{1}{4x^2} + C.$

1.42. $\int x \, \operatorname{arctg} x \, \mathrm{d}x.$ Отг. $\frac{x^2}{2}\operatorname{arctg} x + \frac{1}{2}\operatorname{arctg} x - \frac{x}{2} + C.$

1.43. $\int \ln(x^2+1) \, \mathrm{d}x.$ Отг. $x\ln(x^2+1) + 2\operatorname{arctg} x - 2x + C.$

1.44. $\int \operatorname{arctg} x \, \mathrm{d}x.$ Отг. $x\operatorname{arctg} x - \frac{1}{2}\ln(x^2+1) + C.$

1.45. $\int \frac{1}{x^2}\ln(1+x^2) \, \mathrm{d}x.$ Отг. $-\frac{1}{x}\ln(1+x^2) + 2\operatorname{arctg} x + C.$

1.46. $\int \frac{x}{\cos^2 x} \, \mathrm{d}x.$ Отг. $x\operatorname{tg} x + \ln|\cos x| + C.$

1.47. $\int \frac{\operatorname{arctg} x}{x^2} \, \mathrm{d}x.$ Отг. $-\frac{\operatorname{arctg} x}{x} + \ln|x| - \frac{1}{2}\ln(x^2+1) + C.$

1.48. $\int x \ln\left(x + \frac{1}{x}\right) \, \mathrm{d}x.$ Отг. $\frac{x^2}{4} + \frac{1}{2}x^2\ln\left(x + \frac{1}{x}\right) + \frac{1}{2}\ln(x^2+1) + C.$

1.49. $\int \frac{x + \sqrt{x+2}}{\sqrt{x+2}} \, \mathrm{d}x.$ Отг. $x - 4\sqrt{x+2} + \frac{2}{3}(x+2)^{\frac{3}{2}} + C.$

1.50. $\int \frac{\sqrt{x+1}+2}{\sqrt{x+1}-1} \, \mathrm{d}x.$ Отг. $-6 + x + 6\sqrt{x+1} + 6\ln\left|\sqrt{x+1}-1\right| + C.$

1.51. $\int \frac{\sqrt[3]{x} - 2x}{\sqrt{x}} \, \mathrm{d}x.$ Отг. $\frac{6x^{\frac{5}{6}}}{5} - \frac{4x^{\frac{3}{2}}}{3} + C.$

1.52. $\int \frac{1-\sqrt{x}}{\sqrt{x}(x+1)} \, \mathrm{d}x.$ Отг. $2\operatorname{arctg}\sqrt{x} - \ln|1+x| + C.$

1.53. $\int \frac{1}{\sqrt{x}+\sqrt[3]{x}}\,\mathrm{d}x.$ Отг. $6x^{\frac{1}{6}} - 3x^{\frac{1}{3}} + 2\sqrt{x} - 6\ln(1+x^{\frac{1}{6}}) + C.$

1.54. $\int \operatorname{arctg}\sqrt{x}\,\mathrm{d}x.$ Отг. $-\sqrt{x} + (1+x)\operatorname{arctg}\sqrt{x} + C.$

1.55. $\int \frac{2x+1}{(x-2)(x+4)}\,\mathrm{d}x.$ Отг. $\frac{5}{6}\ln|x-2| + \frac{7}{6}\ln|x+4| + C.$

1.56. $\int \frac{\mathrm{d}x}{(x-2)(x^2+1)}\,.$ Отг. $-\frac{2}{5}\operatorname{arctg} x + \frac{1}{5}\ln|x-2| - \frac{1}{10}\ln(x^2+1) + C.$

1.57. $\int \frac{4x^2-10}{(x-2)(x^2-1)}\,\mathrm{d}x.$ Отг. $2\ln|x-2| + 3\ln|x-1| - \ln|x+1| + C.$

1.58. $\int \frac{\mathrm{d}x}{x^3+3x}.$ Отг. $\frac{1}{3}\ln|x| - \frac{1}{6}\ln(x^2+3) + C.$

1.59. $\int \frac{2x^2+x}{(x-2)(x^2+1)}\,\mathrm{d}x.$ Отг. $\operatorname{arctg} x + 2\ln|x-2| + C.$

1.60. $\int \frac{1}{(x+2)(x+1)^2}\,\mathrm{d}x.$ Отг. $-\frac{1}{x+1} - \ln|x+1| + \ln|x+2| + C.$

1.61. $\int \frac{x^2-2x+2}{x^3-3x^2+2x}\,\mathrm{d}x.$ Отг. $\ln|x-2| - \ln|x-1| + \ln|x| + C.$

2. Да се пресметнат определените интеграли:

2.1. $\int_{-1}^{0} \left(2x^2+3x+5\right).\left(x^2-1\right)\,\mathrm{d}x.$ Отг. $-\frac{57}{20}.$

2.2. $\int_{1}^{3} \left(\frac{2}{x^2} - \frac{3}{2x^3} + \frac{3}{2x^4}\right)\,\mathrm{d}x.$ Отг. $\frac{31}{27}.$

2.3. $\int_{1}^{4} \left(3\sqrt{x} - \frac{1}{\sqrt{x}} + \frac{2}{x^3} + \frac{3}{x^2}\right)\,\mathrm{d}x.$ Отг. $\frac{243}{16}.$

2.4. $\int_{1}^{4} \left(3\sqrt{x} - 2x\right)^2\,\mathrm{d}x.$ Отг. $\frac{27}{10}.$

2.5. $\int_{0}^{1} \left(1+\sqrt{x}\right)^3\,\mathrm{d}x.$ Отг. $\frac{49}{10}.$

2.6. $\int_{0}^{8} \sqrt[3]{7x+8}\,\mathrm{d}x.$ Отг. $\frac{180}{7}.$

2.7. $\int_{1}^{5} \frac{1}{\sqrt{2x-1}}\,\mathrm{d}x.$ Отг. 2.

2.8. $\displaystyle\int_{-1}^{0} \frac{2}{\sqrt[3]{8+7x}}\,\mathrm{d}x.$ Отг. $\frac{9}{7}$.

2.9. $\displaystyle\int_{0}^{3} \sqrt{x+1}\mathrm{d}x + \int_{-2}^{0} \sqrt{1-4x}\,\mathrm{d}x.$ Отг. 9.

2.10. $\displaystyle\int_{0}^{\frac{\pi}{6}} \left(\frac{1}{\cos^2(x+\frac{\pi}{6})} + \frac{1}{\sin^2(x+\frac{\pi}{6})}\right)\,\mathrm{d}x.$ Отг. $\frac{4\sqrt{3}}{3}$.

2.11. $\displaystyle\int_{0}^{\frac{\pi}{2}} \cos^2 x \sin^2 x\,\mathrm{d}x.$ Отг. $\frac{\pi}{8}$.

2.12. $\displaystyle\int_{0}^{2} \frac{2x}{(x^2+2)^2}\,\mathrm{d}x.$ Отг. $\frac{1}{3}$.

2.13. $\displaystyle\int_{0}^{4} \frac{y}{\sqrt{y^2+9}}\,\mathrm{d}y.$ Отг. 2.

2.14. $\displaystyle\int_{0}^{\frac{\pi}{2}} \frac{\cos x}{1+\sin x}\,\mathrm{d}x.$ Отг. $\ln 2$.

2.15. $\displaystyle\int_{0}^{\frac{\pi}{2}} \sin x \cos^2 x\,\mathrm{d}x.$ Отг. $\frac{1}{3}$.

2.16. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{ctg} x\,\mathrm{d}x.$ Отг. $\frac{1}{2}\ln 2$.

2.17. $\displaystyle\int_{1}^{\mathrm{e}} \frac{1+\ln x}{x}\,\mathrm{d}x.$ Отг. $\frac{3}{2}$.

2.18. $\displaystyle\int_{1}^{\mathrm{e}^3} \frac{1}{x(\ln x+2)}\,\mathrm{d}x.$ Отг. $\ln\frac{5}{2}$.

2.19. $\displaystyle\int_{1}^{\mathrm{e}^3} \frac{1}{x\sqrt{1+\ln x}}\,\mathrm{d}x.$ Отг. 2.

2.20. $\displaystyle\int_{0}^{1} \frac{(\operatorname{arctg} x)^2}{1+x^2}\,\mathrm{d}x.$ Отг. $\frac{\pi^3}{192}$.

2.21. $\displaystyle\int_{0}^{1} x\mathrm{e}^x\,\mathrm{d}x.$ Отг. 1.

2.22. $\displaystyle\int_{0}^{\frac{\pi}{2}} x\sin 2x\,\mathrm{d}x.$ Отг. $\frac{\pi}{4}$.

2.23. $\displaystyle\int_{0}^{\frac{\pi}{2}} (x+1)\cos 2x\,\mathrm{d}x.$ Отг. $-\frac{1}{2}$.

2.24. $\int_0^{\frac{\pi}{2}} (3x+1)\sin x\ \mathrm{d}x.$ Отв. 4.

2.25. $\int_0^{\pi} (2x+1)\cos x\ \mathrm{d}x.$ Отв. -4.

2.26. $\int_0^{1} x\mathrm{e}^{-x}\ \mathrm{d}x.$ Отв. $1-\frac{2}{\mathrm{e}}$.

2.27. $\int_0^{1} (2x+1)\mathrm{e}^{x}\ \mathrm{d}x.$ Отв. $\mathrm{e}+1$.

2.28. $\int_1^{\mathrm{e}} x^2\ln x\ \mathrm{d}x.$ Отв. $\frac{1}{9}(1+2\mathrm{e}^3)$.

2.29. $\int_1^{\mathrm{e}} \frac{\ln x}{x^2}\ \mathrm{d}x.$ Отв. $1-\frac{2}{\mathrm{e}}$.

2.30. $\int_0^{1} \operatorname{arctg} x\ \mathrm{d}x.$ Отв. $\frac{\pi}{4}-\frac{1}{2}\ln 2$.

2.31. $\int_0^{1} x^2\operatorname{arctg} x\ \mathrm{d}x.$ Отв. $\frac{1}{12}(-2+\pi+\ln 4)$.

2.32. $\int_0^{\pi} x^3\sin x\ \mathrm{d}x.$ Отв. $\pi^3-6\pi$.

2.33. $\int_2^{7} \frac{x\ \mathrm{d}x}{\sqrt{2+x}}.$ Отв. $\frac{26}{3}$.

2.34. $\int_0^{5} \frac{x-\sqrt{x+4}}{\sqrt{x+4}}\mathrm{d}x.$ Отв. $-\frac{1}{3}$.

2.35. $\int_3^{8} \frac{\sqrt{x+1}+2}{\sqrt{x+1}-1}\mathrm{d}x.$ Отв. $11+6\ln 2$.

2.36. $\int_1^{64} \frac{\sqrt[3]{x}-2}{\sqrt{x}}\mathrm{d}x.$ Отв. $\frac{46}{5}$.

2.37. $\int_0^{2} \frac{1}{\sqrt{x+1}+\sqrt{(x+1)^3}}\mathrm{d}x.$ Отв. $\frac{\pi}{6}$.

2.38. $\int_0^{\ln 5} \frac{\mathrm{e}^x\sqrt{\mathrm{e}^x-1}}{\mathrm{e}^x+3}\mathrm{d}x.$ Отв. $4-\pi$.

2.39. $\int_0^{1} \frac{x-1}{(x+2)(x+1)}\mathrm{d}x.$ Отв. $\ln\frac{27}{32}$.

2.40. $\int_0^{2} \frac{3x+1}{(x+2)(x+1)}\mathrm{d}x.$ Отв. $\ln\frac{32}{9}$.

2.41. $\int_1^4 \frac{1}{x^3+3x}\mathrm{d}x$. Отг. $\frac{1}{2}\ln\frac{64}{19}$.

2.42. $\int_3^4 \frac{x^2-2x+2}{x^3-3x^2+2x}\mathrm{d}x$. Отг. $\ln\frac{16}{9}$.

2.43. $\int_0^2 \frac{\mathrm{d}x}{\sqrt{x+1}(x+2)}$. Отг. $\frac{\pi}{6}$.

3. Да се намери лицето на фигурата, ограничена от линиите, зададени с уравненията:

3.1. $y = x^2 + 3,\ y = 0,\ x = -1,\ x = 2.$ Отг. 12.

3.2. $y = x^2 - 3x,\ x + y = 0.$ Отг. $\frac{4}{3}$.

3.3. $y = x^2 + 4x,\ x - y + 4 = 0.$ Отг. $\frac{125}{6}$.

3.4. $y = x^2 - 3,\ y = 6x - x^2 - 7.$ Отг. $\frac{1}{3}$.

3.5. $y = 2x^2 + 10,\ y = 22 - x^2.$ Отг. 32.

3.6. $y = x^2,\ x = y^2.$ Отг. $\frac{1}{3}$.

3.7. $4y = x^2,\ y^2 = 4x.$ Отг. $\frac{16}{3}$.

3.8. $y = x\mathrm{e}^{2x},\ y = 0, 0 \le x \le 1.$ Отг. $\frac{1}{4}\left(1 + \mathrm{e}^2\right)$.

3.9. $y = 4x\sin 2x, y = 0, 0 \le x \le \frac{\pi}{2}$. Отг. π.

3.10. $x^2 + y^2 = 16,\ y^2 = 6x.$ Отг. $\frac{16\pi}{3} + \frac{4\sqrt{3}}{3}$; $\frac{32\pi}{3} - \frac{4\sqrt{3}}{3}$.

3.11. $y = \mathrm{e}^x,\ y = \mathrm{e}^{-x}, x = 1.$ Отг. $\mathrm{e} + \mathrm{e}^{-1} - 2$.

3.12. $y = \ln x,\ y = \ln^2 x.$ Отг. $3 - \mathrm{e}$.

3.13. $\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t), \ \ 0 \le t \le 2\pi, a > 0, y = 0. \end{cases}$ Отг. $3a^2\pi$.

3.14. $\begin{cases} x = 3\cos t + 5\sin t \\ y = 5\cos t - 3\sin t, \ \ 0 \le t \le 2\pi. \end{cases}$ Отг. 34π.

3.15. $\begin{cases} x = a\cos^3 t \\ y = a\sin^3 t, \ \ 0 \le t \le 2\pi, a > 0. \end{cases}$ Отг. $\frac{3a^2\pi}{8}$.

3.16. $\rho = \sin\theta + \cos\theta,\ -\frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$. Отг. $\frac{\pi}{2}$.

3.17. $\rho = a\sin 3\theta,\ 0 \le \theta \le \frac{\pi}{3},\ a > 0.$ Отг. $\frac{a^2\pi}{12}$.

4. Да се намери дължината на линията, зададена с уравнения:

4.1. $y = 4x^{\frac{3}{2}},\ 0 \le x \le \frac{2}{3}$. Отг. $\frac{62}{27}$.

4.2. $y = x^{\frac{3}{2}},\ 0 \le x \le \frac{20}{3}$. Отг. $\frac{56}{3}$.

4.3. $y = \ln x,\ \sqrt{3} \le x \le \sqrt{8}$. Отг. $1 + \frac{1}{2}\ln\frac{3}{2}$.

4.4. $y = \sqrt{\mathrm{e}^{2x} - 1} - \operatorname{arctg}\sqrt{\mathrm{e}^{2x} - 1},\ 0 \le x \le 1$. Отг. $\mathrm{e} - 1$.

4.5. $y = \sqrt{x - x^2} - \arcsin\sqrt{x},\ 0 \le x \le 1$. Отг. 2.

4.6. $y = \ln\cos x,\ 0 \le x \le \frac{\pi}{3}$. Отг. $\ln\left(\operatorname{tg}\frac{5\pi}{12}\right)$.

4.7. $y = \ln(1 - x^2),\ 0 \le x \le \frac{1}{2}$. Отг. $\ln 3 - \frac{1}{2}$.

4.8. $\begin{cases} x = 5 + 3\cos 2t \\ y = 5 + 3\sin 2t, \ \ 0 \le t \le \frac{\pi}{2}. \end{cases}$ Отг. 3π.

4.9. $\begin{cases} x = 6\cos t + 8\sin t \\ y = 8\cos t - 6\sin t, \ \ 0 \le t \le \pi. \end{cases}$ Отг. 10π.

4.10. $\begin{cases} x = \cos 2t + \sin 2t \\ y = \sin 2t - \cos 2t, \ \ 0 \le t \le \frac{\pi}{2}. \end{cases}$ Отг. $\sqrt{2}\pi$.

4.11. $\begin{cases} x = \sqrt{5}\sin 3t - 2\cos 3t \\ y = \sqrt{5}\cos 3t + 2\sin 3t, \ \ 0 \le t \le \frac{\pi}{3}. \end{cases}$ Отг. 3π.

4.12. $\begin{cases} x = 2(t - \sin t) \\ y = 2(1 - \cos t), \ \ 0 \le t \le \pi. \end{cases}$ Отг. 8.

4.13. $\begin{cases} x = \mathrm{e}^t\cos t \\ y = \mathrm{e}^t\sin t, \ \ 0 \le t \le 1. \end{cases}$ Отг. $\sqrt{2}(\mathrm{e} - 1)$.

4.14. $\begin{cases} x = \mathrm{e}^t(\sin t + \cos t) \\ y = \mathrm{e}^t(\cos t - \sin t), \ \ 0 \le t \le \ln 2. \end{cases}$ Отг. 2.

4.15. $\begin{cases} x = 3(\cos t + t\sin t) \\ y = 3(\sin t - t\cos t), \ \ 0 \le t \le \pi. \end{cases}$ Отг. $\frac{3\pi^2}{2}$.

4.16. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t \\ y = (2 - t^2)\cos t + 2t\sin t, \ \ 0 \le t \le \pi. \end{cases}$ Отг. $\frac{\pi^3}{3}$.

4.17. $\begin{cases} x = a\cos^3 t \\ y = a\sin^3 t, \ \ 0 \le t \le \frac{\pi}{2}. \end{cases}$ Отг. $\frac{3a}{2}$.

4.18. $\rho = a\,(1 + \cos\theta)\,,\ 0 \le \theta \le 2\pi,\ a > 0$. Отг. $8a$.

4.19. $\rho = \sin^3\dfrac{\theta}{3},\ 0 \le \theta \le 3\pi$. Отг. $\frac{3\pi}{2}$.

5. Да се намерят частните производни от първи ред на следните функции:

5.1. $f(x,y) = 5x^3y^3 + 2\mathrm{e}^{3y} + 2\ln 4x.$

Отг. $f'_x = 15x^2y^3 + \frac{2}{x};\ f'_y = 15x^3y^2 + 6\mathrm{e}^{3y}.$

5.2. $f(x,y) = \mathrm{e}^y(2 + x + y^2).$

Отг. $f'_x = \mathrm{e}^y;\ f'_y = \mathrm{e}^y(2 + x + y^2 + 2y).$

5.3. $f(x,y) = \mathrm{e}^{2x}(1 - x^2 - y^2).$

Отг. $f'_x = 2\mathrm{e}^{2x}(1 - x^2 - y^2 - x);\ f'_y = -2y\mathrm{e}^{2x}.$

5.4. $f(x,y) = \ln(x^2 + y^2) + \mathrm{e}^{2x+3y}.$

Отг. $f'_x = \frac{2x}{x^2+y^2} + 2\mathrm{e}^{2x+3y};\ f'_y = \frac{2y}{x^2+y^2} + 3\mathrm{e}^{2x+3y}.$

5.5. $f(x,y) = 2xy\mathrm{e}^{x+2y}.$

Отг. $f'_x = 2y\mathrm{e}^{x+2y}(1 + x);\ f'_y = 2x\mathrm{e}^{x+2y}(1 + 2y).$

5.6. $f(x,y) = \dfrac{2xy}{x^2 + y^2}.$

Отг. $f'_x = \frac{2y(y^2-x^2)}{(x^2+y^2)^2};\ f'_y = \frac{2x(x^2-y^2)}{(x^2+y^2)^2}.$

5.7. $f(x,y) = \ln\left(x + \sqrt{x^2 + y^2}\right).$

Отг. $f'_x = \frac{1}{\sqrt{x^2+y^2}};\ f'_y = \frac{y}{x^2+y^2+x\sqrt{x^2+y^2}}.$

5.8. $f(x,y) = \operatorname{arctg}\dfrac{x}{y}.$ Отг. $f'_x = \frac{y}{x^2+y^2};\ f'_y = \frac{-x}{x^2+y^2}.$

5.9. $f(x,y) = \operatorname{arctg}\dfrac{x+y}{1-xy}.$ Отг. $f'_x = \frac{1}{1+x^2};\ f'_y = \frac{1}{1+y^2}.$

6. Да се проверят следните равенства:

6.1. $z'_x + z'_y = 2,$ ако $z(x,y) = \ln\left(\mathrm{e}^{2x} + \mathrm{e}^{2y}\right).$

6.2. $\dfrac{1}{x}.z'_x + \dfrac{1}{y}.z'_y = \dfrac{z}{y^2},$ ако $z(x,y) = \dfrac{y}{x^2 - y^2}.$

6.3. $x.z'_x + y.z'_y = z,$ ако $z(x,y) = x\ln\dfrac{y}{x}.$

6.4. $x.z'_x + y.z'_y = 2xy\sqrt{1 - z^2},$ ако $z(x,y) = \sin\left(xy + \dfrac{y}{x}\right).$

6.5. $x.z'_x + y.z'_y = 0,$ ако $z(x,y) = \operatorname{arctg}\dfrac{x}{y}.$

7. Да се намерят вторите частни производни на следните функции:

7.1. $z(x,y) = x^3 + xy^2 - 5xy^3 + y^5$.

Отг. $z''_{xx} = 6x;\ z''_{xy} = 2y - 15y^2;\ z''_{yy} = 2x - 30xy + 20y^3$.

7.2. $z(x,y) = \ln\left(x^2 + y^2\right)$.

Отг. $z''_{xx} = \frac{2\left(y^2-x^2\right)}{\left(x^2+y^2\right)^2};\ z''_{xy} = -\frac{4xy}{\left(x^2+y^2\right)^2};\ z''_{yy} = \frac{2\left(x^2-y^2\right)}{\left(x^2+y^2\right)^2}$.

7.3. $z(x,y) = x\mathrm{e}^{x-2y}$.

Отг. $z''_{xx} = (2+x)\mathrm{e}^{x-2y};\ z''_{xy} = -2(x+1)\mathrm{e}^{x-2y};\ z''_{yy} = 4x\mathrm{e}^{x-2y}$.

7.4. $z(x,y) = \mathrm{e}^{-3x} + \sin 2y + \ln(x+y)$.

Отг. $z''_{xx} = 9\mathrm{e}^{-3x} - \frac{1}{(x+y)^2};\ z''_{xy} = -\frac{1}{(x+y)^2};\ z''_{yy} = -4\sin 2y - \frac{1}{(x+y)^2}$.

8. Да се проверят следните равенства:

8.1. $3z''_{xx} + z''_{yy} + z''_{xy} = 2x$, ако $z(x,y) = 3xy^2 - 2x^2y + y^3$.

8.2. $yz'_x + xz'_y - z''_{xy} = 0$, ако $z(x,y) = \mathrm{e}^{x^2+y^2}$.

8.3. $z''_{xy} - 2yz'_x = 0$, ако $z(x,y) = 2x\mathrm{e}^{y^2-x^2}$.

8.4. $z''_{yy} = 4z''_{xx}$, ако $z(x,y) = \sin(x-2y) + \cos(x+2y)$.

8.5. $4z''_{yy} = 9z''_{xx}$, ако $z(x,y) = \mathrm{e}^{2x-3y} + \mathrm{e}^{2x+3y}$.

8.6. $z''_{xx} + z''_{yy} = 0$, ако $z(x,y) = \ln\dfrac{1}{\sqrt{x^2+y^2}}$.

8.7. $xz''_{xx} + yz''_{xy} = 0$, ако $z(x,y) = \sqrt{x^2+2xy}$.

8.8. $z''_{xx} + z''_{yy} = 0$, ако $z(x,y) = \operatorname{arctg}\dfrac{y}{x}$.

9. Да се намерят локалните екстремуми на следните функции:

9.1. $f(x,y) = x^2 + y^2 - 4x - 2y + 1$. Отг. $z_{\min}(2,1) = -4$.

9.2. $f(x,y) = 6(x-y) - x^2 - y^2$. Отг. $z_{\max}(3,-3) = 18$.

9.3. $f(x,y) = x^2 + y^2 + xy + x + y$. Отг. $z_{\min}\left(-\frac{1}{3},-\frac{1}{3}\right) = -\frac{1}{3}$.

9.4. $f(x,y) = x^2 + xy + y^2 - 6x - 9y$. Отг. $z_{\min}(1,4) = -21$.

9.5. $f(x,y) = x^2 + y^3 - 4x - 12y + 1$. Отг. $z_{\min}(2,2) = -19$.

9.6. $f(x,y) = x^3 - \frac{3}{2}y^2 - 12x + 12y + 30$. Отг. $z_{\max}(-2,4) = 70$.

9.7. $f(x,y) = x^2 + y^3 - 2x - 3y + 1.$ Отг. $z_{\min}(1,1) = -2.$

9.8. $f(x,y) = x^3 + y^2 - 6xy - 39x + 18y.$ Отг. $z_{\min}(5,6) = -106.$

9.9. $f(x,y) = x^3 + y^3 - 6xy.$ Отг. $z_{\min}(2,2) = -8.$

9.10. $f(x,y) = x^3 + 8y^3 - 6xy + 3.$ Отг. $z_{\min}\left(1, \frac{1}{2}\right) = 2.$

9.11. $f(x,y) = x^3 + y^3 - 3x - 3y.$ Отг. $z_{\max}(-1,-1) = 4, z_{\min}(1,1) = -4.$

9.12. $f(x,y) = \mathrm{e}^x\left(x^2 + y^2 - 3\right).$ Отг. $z_{\min}(1,0) = -2\mathrm{e}.$

9.13. $f(x,y) = \mathrm{e}^{2x}\left(x + y^2 + 2y\right).$ Отг. $z_{\min}\left(\frac{1}{2}, -1\right) = -\frac{\mathrm{e}}{2}.$

9.14. $f(x,y) = \mathrm{e}^y\left(y + x^3 - 3x\right).$ Отг. $z_{\min}(1,1) = -\mathrm{e}.$

9.15. $f(x,y) = \mathrm{e}^{x-y}\left(x^2 - 2y^2\right).$ Отг. $z_{\min}(-4,-2) = \frac{8}{\mathrm{e}^2}.$

10. Решете диференциалните уравнения с отделящи се променливи:

10.1. $3x^2\mathrm{d}x + (2y+1)\mathrm{d}y = 0.$ Отг. $x^3 + y^2 + y = c.$

10.2. $x(1+y^2)\mathrm{d}x + y(1+x^2)\mathrm{d}y = 0.$ Отг. $(1+x^2)(1+y^2) = c.$

10.3. $3x^2(y+1)\mathrm{d}x - (x^3+1)\mathrm{d}y = 0.$ Отг. $\frac{x^3+1}{y+1} = c.$

10.4. $(x+1)y\mathrm{d}x + (1-y)x\mathrm{d}y = 0.$ Отг. $\ln|xy| + x - y = c.$

10.5. $\sqrt{1-y^2}\mathrm{d}x + y\sqrt{1-x^2}\mathrm{d}y = 0.$ Отг. $\arcsin x - \sqrt{1-y^2} = c.$

10.6. $y\,\mathrm{e}^{2x}\mathrm{d}x - (1+\mathrm{e}^{2x})\mathrm{d}y = 0.$ Отг. $y = c\sqrt{1+\mathrm{e}^{2x}}.$

10.7. $x(1+y^2)\mathrm{d}x + \sqrt{1+x^2}\mathrm{d}y = 0.$ Отг. $\sqrt{1+x^2} + \operatorname{arctg} y = c.$

10.8. $\sqrt{y^2+1}\mathrm{d}x - xy\mathrm{d}y = 0.$ Отг. $\ln|x| - \sqrt{y^2+1} = c.$

10.9. $\dfrac{\operatorname{tg} y}{\cos^2 x}\mathrm{d}x + \dfrac{\operatorname{tg} x}{\cos^2 y}\mathrm{d}y = 0.$ Отг. $\operatorname{tg} x \operatorname{tg} y = c.$

10.10. $(1+x^2)y^3\mathrm{d}x - (y^2-1)x^3\mathrm{d}y = 0, y(1) = -1.$ Отг. $\frac{1}{x^2} + \frac{1}{y^2} = 2\left(1 + \ln\left|\frac{x}{y}\right|\right).$

10.11. $y' = \dfrac{1-2x}{y^2}.$ Отг. $y^3 = 3(x - x^2 + c).$

10.12. $y'\sqrt{1-x^2} = 1 + y^2.$ Отг. $\operatorname{arctg} y = \arcsin x + C.$

10.13. $y'\sin x = y\ln y.$ Отг. $y = \mathrm{e}^{c\,tg\frac{x}{2}}.$

10.14. $y'y^3 = 1 - 2x.$ Отг. $y^4 = 4(x - x^2 + c).$

10.15. $y' = \dfrac{y^2+y}{x-x^2}.$ Отг. $y(1-x) = cx(1+y).$

10.16. $xy' + y = y^2,\ y(1) = \dfrac{1}{2}.$ Отг. $y = \frac{1}{1+x}.$

10.17. $x^2(2yy' - 1) = 1.$ Отг. $x(y^2 + c) = x^2 - 1.$

10.18. $2(1 + \mathrm{e}^x)yy' = \mathrm{e}^x,\ y(0) = 0.$ Отг. $2\mathrm{e}^{y^2} = \mathrm{e}^x + 1.$

11. Решете хомогенните диференциални уравнения от първи ред:

11.1. $xy' = y \ln \frac{y}{x}.$ Отг. $y = x\mathrm{e}^{cx+1}.$

11.2. $y' = \frac{y}{x}\left(1 - \ln \frac{y}{x}\right).$ Отг. $y = x\,\mathrm{e}^{\frac{c}{x}}.$

11.3. $y' = \mathrm{e}^{\frac{y}{x}} + \frac{y}{x}.$ Отг. $\ln|x| + \mathrm{e}^{-\frac{y}{x}} = c.$

11.4. $y'\left(1 - \frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 = 0.$ Отг. $\ln y = \frac{y}{x} + c,\ y = 0.$

11.5. $y' = \sin \frac{y}{x} + \frac{y}{x}.$ Отг. $\mathrm{tg}\,\frac{y}{2x} = cx,\ y = 0.$

11.6. $y' = \frac{x + y}{x - y}.$ Отг. $\mathrm{arctg}\,\frac{y}{x} = \ln\left|c\sqrt{x^2 + y^2}\right|.$

11.7. $(xy' - y)\,\mathrm{arctg}\,\frac{y}{x} = x,\ y(1) = 0.$ Отг. $\sqrt{x^2 + y^2} = \mathrm{e}^{\frac{y}{x}\,\mathrm{arctg}\,\frac{y}{x}}.$

11.8. $\left(x^2 + y^2\right)y' = 2xy.$ Отг. $y^2 - x^2 = cy, y = 0.$

11.9. $(y - xy')\cos \frac{y}{x} = x.$ Отг. $\sin \frac{y}{x} + \ln|x| = c.$

12. Решете линейните диференциални уравнения от първи ред:

12.1. $y' + \frac{3}{x}y = x^2.$ Отг. $y = \frac{x^3}{6} + \frac{c}{x^3}.$

12.2. $y' - \frac{3}{x}y = x^2.$ Отг. $y = c\,x^3 + x^3\,\ln|x|.$

12.3. $y' = x^2 - \frac{y}{x}.$ Отг. $y = \frac{x^3}{4} + \frac{c}{x}.$

12.4. $y' + y = \mathrm{e}^{-x}.$ Отг. $y = (x + c)\mathrm{e}^{-x}.$

12.5. $y' + 2y\ \mathrm{tg}\,x = \cos^2 x.$ Отг. $y = \cos^2 x\,(c + x).$

12.6. $y' + y\ \mathrm{tg}\,x = \frac{1}{\cos x}.$ Отг. $y = \sin x + c\cos x.$

12.7. $y' - 2\mathrm{cotg}x\,y = \sin^3 x.$ Отг. $y = \sin^2 x\,(c - \cos x).$

12.8. $y' - y\,\mathrm{cotg}x = \sin x.$ Отг. $y = \sin x\,(c + x).$

12.9. $y' - 2xy = \left(x^3 - x\right)\,\mathrm{e}^{x^2}.$ Отг. $y = \mathrm{e}^{x^2}\left(c + \frac{x^4}{4} - \frac{x^2}{2}\right).$

12.10. $y' - 3x^2 y = x^2 \mathrm{e}^{x^3}, y(0) = 2.$ Отг. $y = \mathrm{e}^{x^3}\left(2 + \frac{x^3}{3}\right)\cdot$

12.11. $y' + 2xy = 2x^2 \mathrm{e}^{-x}.$ Отг. $y = \mathrm{e}^{-x^2}\left(c + \frac{2x^3}{3}\right)\cdot$

12.12. $(1+x^2)y' = 2xy + (1+x^2)^2.$ Отг. $y = (1+x^2)(x+c).$

12.13. $x\,y' + y - \mathrm{e}^x = 0.$ Отг. $y = \frac{1}{x}\left(c + \mathrm{e}^x\right).$

12.14. $y' + \dfrac{1-2x}{x^2}\,y = 1.$ Отг. $y = c\,x^2 \mathrm{e}^{\frac{1}{x}} + x^2.$

12.15. $y' + \dfrac{x+1}{x}\,y = 3x\,\mathrm{e}^{-x}.$ Отг. $y = \dfrac{c + x^3}{x\,\mathrm{e}^x}\cdot$

12.16. $y' - \dfrac{2y}{x+1} = (x+1)^{\frac{5}{2}}.$ Отг. $y = (x+1)^2\left(c + \frac{2(x+1)^{\frac{3}{2}}}{3}\right).$

12.17. $y' + 2y = 4x.$ Отг. $y = 2x - 1 + c\,\mathrm{e}^{-2x}.$

12.18. $y' + 3y = 6x.$ Отг. $y = 2x - \frac{2}{3} + c\,\mathrm{e}^{-3x}.$

12.19. $x\,y' - \dfrac{y}{x+1} = x,\ y(1) = 0.$ Отг. $y = \frac{x}{x+1}\left(x + \ln|x| - 1\right)\cdot$

12.20. $x\,y' + y = x + 1,\ y(2) = 3.$ Отг. $y = \dfrac{2}{x} + \dfrac{x}{2} + 1.$

12.21. $y = x\,(y' - x\cos x).$ Отг. $y = x\,(c + \sin x)\,.$

12.22. $y' = \cos x - y.$ Отг. $y = c\,\mathrm{e}^{-x} + \frac{1}{2}(\sin x + \cos x).$

12.23. $y' + y\cos x = \dfrac{1}{2}\sin 2x.$ Отг. $y = \sin x - 1 + c\,\mathrm{e}^{-\sin x}.$

12.24. $y' = \dfrac{y}{3x - y^2}\cdot$ Отг. $x = c\,y^3 + y^2.$

12.25. $y' = \dfrac{y}{4x + 2y}\cdot$ Отг. $x = c\,y^4 - \frac{2}{3}y.$

12.26. $y' = \dfrac{1}{2x - y^2}\cdot$ Отг. $x = c\,\mathrm{e}^{2y} + \frac{1}{2}y^2 + \frac{1}{2}y + \frac{1}{4}\cdot$

12.27. $y'\,(x + y^2) = -1.$ Отг. $x = c\,\mathrm{e}^{-y} + 2y - y^2 - 2.$

12.28. $y' = \dfrac{y}{2y\ln y + y - x}\cdot$ Отг. $x = \dfrac{c}{y} + y\,\ln y.$

13. Решете диференциалните уравнения на Бернули:

13.1. $y' = y\,\mathrm{tg}x + y^4\cos x.$ Отг. $y = \left[\cos^3 x\ (c - 3tgx)\right]^{-\frac{1}{3}}.$

13.2. $y' = y\,\mathrm{cotg}x + \dfrac{y^3}{\sin x}\cdot$ Отг. $y = \dfrac{\sin x}{\sqrt{c + 2\cos x}}\cdot$

13.3. $y' x + y = y^2 \ln x$. Отг. $y = (c\,x + \ln|x| + 1)^{-1}$.

13.4. $y' + x\,y = x^3 y^3$. Отг. $y = \left(c\,\mathrm{e}^{x^2} + x^2 + 1\right)^{-\frac{1}{2}}$.

13.5. $y' - \dfrac{y}{x} = \dfrac{x}{y^2}$. Отг. $y = \sqrt[3]{c\,x^3 - 3x^2}$.

13.6. $x\,y' - 4y = x^2\sqrt{y}$. Отг. $y = x^4\left(c + \frac{1}{2}\ln|x|\right)^2$.

13.7. $y' + y = x\,\sqrt{y}$. Отг. $y = \left(c\,\mathrm{e}^{-\frac{x^2}{2}} + x - 2\right)^2$.

13.8. $y' + 4xy = 2x\,\mathrm{e}^{-x^2}\sqrt{y}$. Отг. $y = \mathrm{e}^{-2x^2}\left(c + \frac{x^2}{2}\right)^2$.

14. Решете тоталните диференциални уравнения:

14.1. $(9x - 10y)\ \mathrm{d}x + (3y - 10x)\ \mathrm{d}y = 0$. Отг. $\frac{9x^2}{2} - 10xy + \frac{3y^2}{2} = c$.

14.2. $\left(3x^2 + 3y\right)\ \mathrm{d}x + (3x + y)\ \mathrm{d}y = 0$. Отг. $x^3 + 3xy + \frac{y^2}{2} = c$.

14.3. $\left(3x^2y + 3xy^2 + 2\right)\ \mathrm{d}x + \left(x^3 + 3x^2y\right)\ \mathrm{d}y = 0$. Отг. $x^3y + \frac{3}{2}x^2y^2 + 2x = c$.

14.4. $\mathrm{e}^y\mathrm{d}x + (x\mathrm{e}^y - 2y)\ \mathrm{d}y = 0$. Отг. $x\,\mathrm{e}^y - y^2 = c$.

14.5. $(3y - \cos y)\ \mathrm{d}x + (3x + x\sin y)\,\mathrm{d}y = 0$. Отг. $3\,x\,y - x\ \cos y = c$.

14.6. $(2xy + 2\sin y)\ \mathrm{d}x + (x^2 + 2x\cos y)\,\mathrm{d}y = 0$. Отг. $x^2\,y + 2x\ \sin y = c$.

14.7. $(y\mathrm{e}^x - 3x)\ \mathrm{d}x + \mathrm{e}^x\ \mathrm{d}y = 0$. Отг. $y\mathrm{e}^x - \frac{3x^2}{2} = c$.

14.8. $\left(\dfrac{1}{x^2} - y\right)\ \mathrm{d}x + (y - x)\ \mathrm{d}y = 0$. Отг. $xy^2 - 2x^2y - 2 = cx$.

14.9. $\left(2x - 1 - \dfrac{y}{x^2}\right)\ \mathrm{d}x + \left(\dfrac{1}{x} - 2y\right)\ \mathrm{d}y = 0$. Отг. $x^2 - x + \frac{y}{x} - y^2 = c$.

14.10. $2x\cos^2 y\,\mathrm{d}x + \left(2y - x^2\sin 2y\right)\ \mathrm{d}y = 0$. Отг. $x^2\cos^2 y + y^2 = c$.

14.11. $\dfrac{x\,\mathrm{d}x + y\,\mathrm{d}y}{\sqrt{x^2 + y^2}} = \dfrac{y\,\mathrm{d}x - x\,\mathrm{d}y}{x^2}$ Отг. $\sqrt{x^2 + y^2} + \frac{y}{x} = c$.

15. Решете диференциалните уравнения от втори ред:

15.1. $y'' - 3y' + 2y = 2\mathrm{e}^{3x}$. Отг. $y = c_1\mathrm{e}^x + c_2\mathrm{e}^{2x} + \mathrm{e}^{3x}$.

15.2. $y'' - 3y' + 2y = 4\mathrm{e}^x$. Отг. $y = c_1\mathrm{e}^x + c_2\mathrm{e}^{2x} - 4x\mathrm{e}^x$.

15.3. $y'' - 2y' + y = (x + 1)\ \mathrm{e}^{-2x}$. Отг. $y = c_1\mathrm{e}^x + c_2x\mathrm{e}^x + \left(\frac{1}{9}x + \frac{5}{27}\right)\mathrm{e}^{-2x}$.

15.4. $y'' - y = x\,\mathrm{e}^x$. Отг. $y = c_1\mathrm{e}^x + c_2\mathrm{e}^{-x} + \frac{1}{4}\left(x^2 - x\right)\mathrm{e}^x$.

15.5. $y'' + y' = \mathrm{e}^{2x}$. Отг. $y = c_1 + c_2\mathrm{e}^{-x} + \frac{1}{6}\mathrm{e}^{2x}$.

15.6. $y'' + 2y' + y = 2\sin x$. Отг. $y = c_1\mathrm{e}^{-x} + c_2 x\mathrm{e}^{-x} - \cos x$.

15.7. $y'' + 4y = 2\sin 2x$. Отг. $y = c_1\cos 2x + c_2\sin 2x - \frac{x}{2}\cos 2x$.

15.8. $y'' - 4y' + 4y = x^2 + 1$. Отг. $y = c_1\mathrm{e}^{2x} + c_{2v}\mathrm{e}^{2x} + \frac{x^2}{4} + \frac{x}{2} + \frac{5}{8}$.

15.9. $y'' - 4y' + 4y = 13\cos 3x$. Отг. $y = c_1\mathrm{e}^{2x} + c_2 x\mathrm{e}^{2x} - \frac{5}{13}\cos 3x - \frac{12}{13}\sin 3x$.

15.10. $y'' + y = 2x^3 - x + 2$. Отг. $y = c_1\cos x + c_2\sin x + 2x^3 - 13x + 2$.

15.11. $y'' + y = \cos x - 2\mathrm{e}^{-x}$. Отг. $y = c_1\cos x + c_2\sin x + \frac{x}{2}\sin x - \mathrm{e}^{-x}$.

15.12. $y'' + y = \cos 2x - \mathrm{e}^{2x}$. Отг. $y = c_1\cos x + c_2\sin x - \frac{1}{3}\cos 2x - \frac{1}{5}\mathrm{e}^{2x}$.

15.13. $y'' - y' - 6y = (2 - x)\ \mathrm{e}^{3x}$. Отг. $y = c_1\mathrm{e}^{-2x} + c_2\mathrm{e}^{3x} + \left(-\frac{1}{10}x^2 + \frac{11}{25}x\right)\mathrm{e}^{3x}$.

15.14. $y'' - y' = x^2 + 2x - 5$. Отг. $y = c_1 + c_2\mathrm{e}^{x} + x\left(-\frac{x^2}{3} - 2x + 1\right)$.

15.15. $y'' - 4y' + 3y = x^2\mathrm{e}^{2x}$. Отг. $y = c_1\mathrm{e}^{x} + c_2\mathrm{e}^{3x} - (x^2 + 2)\,\mathrm{e}^{2x}$.

15.16. $y'' - 6y' + 5y = x^2\mathrm{e}^{x}$. Отг. $y = c_1\mathrm{e}^{x} + c_2\mathrm{e}^{5x} - x\left(\frac{1}{12}x^2 + \frac{1}{16}x + \frac{1}{32}\right)\mathrm{e}^{x}$.

16. Изследвайте за сходимост числовите редове:

16.1. $\sum\limits_{n=1}^{\infty}\frac{n}{2^n}\cdot$ Отг. Сходящ.

16.2. $\sum\limits_{n=1}^{\infty}\frac{(n+1)}{3^n}\cdot$ Отг. Сходящ.

16.3. $\sum\limits_{n=1}^{\infty}\frac{n^2}{3^n}\cdot$ Отг. Сходящ.

16.4. $\sum\limits_{n=1}^{\infty}\frac{n^2}{n!}$ Отг. Сходящ.

16.5. $\sum\limits_{n=1}^{\infty}\frac{n}{n^2+1}\cdot$ Отг. Разходящ.

16.6. $\sum\limits_{n=1}^{\infty}\frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n}\cdot$ Отг. Сходящ.

16.7. $\sum\limits_{n=2}^{\infty}\frac{n}{2^n}\cdot$ Отг. Сходящ.

16.8. $\sum\limits_{n=1}^{\infty}\frac{n!}{6^n}\cdot$ Отг. Разходящ.

16.9. $\sum\limits_{n=2}^{\infty}\frac{3^n.n!}{n^n}\cdot$ Отг. Разходящ.

16.10. $\sum\limits_{n=1}^{\infty}\frac{(-1)^n n}{2^n}\cdot$ Отг. Сходящ.

16.11. $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}\cdot$ Отг. Сходящ.

16.12. $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n-1)^3}\cdot$ Отг. Сходящ.

16.13. $\sum\limits_{n=1}^{\infty}(-3)^n(n+1)\cdot$ Отг. Разходящ.